Maths Masters summer quiz

by Burkard Polster and Marty Ross

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Welcome to our third Maths Masters Summer Quiz. This year we have thirty puzzles, perfect to keep you entertained on an otherwise lazy day. The puzzles are in three sections: easy, medium, and evilly hard.

They have been gathered from many sources. A number of them appear in the excellent book <u>*Ants*</u>, <u>*Bikes and Clocks*</u> by William Briggs. We hope that you find all of the puzzles enjoyable and engaging.

Answers will appear on <u>our website</u> on December 3, and detailed solutions will appear there on December 6. As always, *all* correspondence will be entered into. If you believe a puzzle is in error, or you disagree with any of our solutions, or feel you have a better solution, please let us know.

Good Luck, and enjoy!

EASY

Easy 1. If five robots can make five new robots in five hours, how long would it take a hundred robots to make a hundred new robots?



Easy 2. Two poker chips are pictured below. The two visible sides add up to 10. Flipping the chips in all possible ways, you get the sums 7, 8, 9, and 10. What are the numbers on the other side of the chips?



Easy 3. A drawer full of socks contains 5 black socks and 6 blues socks. You take socks out of the drawer without looking. How many socks must you take out to be sure of having a matching pair? What if you want to be sure of having a black pair?



Easy 4. The triangle of coins below is pointing upwards. By moving only three coins, arrange for the triangle to point downwards.



Easy 5. Three-dimensional Homer stumbles across the equation $1782^{12} + 1841^{12} = 1922^{12}$. Is the equation true or false? (Put that calculator down!)



Easy 6. Which is larger, 1000/3000 or 1001/3001? (Put that calculator down!)



Easy 7. Julie takes a third of the jellybeans from a large jar. Then Tony takes a third of the remaining jellybeans from the jar. Finally, Bob takes a third of the remaining jellybeans, leaving 40 jellybeans in the jar. How many jellybeans were there to begin with?



Easy 8. A bat and a ball together cost \$110. The bat costs \$100 more than the ball. How much does the ball cost?



Easy 9. Three tennis balls fit exactly inside a cylindrical can. Which is greater, the circumference of the can or the length of the can? Which is greater, the combined area of the three balls, or the area of the cylinder (excluding the base and the lid)?



Easy 10. Renae walks into an electronics store to purchase a \$1000 television, and she has three coupons. One coupon entitles Renae to a 50% discount, the second to a 30% discount, and the third to a 20% discount. David the salesmen agrees to let Renae use all three coupons at once. How much does Renae pay for the television?



MEDIUM

Medium 1. Kylie and Dannii ran a 100 metres race, and Kylie won by 5 metres. They plan to run a second race, with Kylie beginning 5 metres behind the starting line. Assuming both run the same speed as in the first race, who will win?



Medium 2. A litre of paint is required to paint a 4 metre high statue of QEDcat. How many litres do you need to paint 50 statuettes of QEDcat, each 20 centimetres high?



octagon inside it, as pictured. Now draw a regular 16-sided polygon inside the octagon, and so on. What is the area trapped inside all the polygons?



Medium 4. A forest outside Melbourne is inhabited by wild QEDcats. One day you catch 30 QEDcats. You tag their ears, and set them free. The next day you catch 20 QEDcats, of which 2 have tagged ears. How many QEDcats do you estimate inhabit the forest?



Medium 5. You have ten large sacks of coins, but one of the sacks contains counterfeit coins. The real coins weigh 20 grams each and the counterfeit coins weigh 15 grams each. You have a scale with which you can weigh any number of coins. Using just one weighing, how can you tell which is the sack of counterfeit coins?



Medium 6. Replace the letters Q, E, and D by different digits to make the following equation true.

$QED = (Q + E + D) \times Q \times E \times D$

Medium 7. You want to measure the longest diagonal of a brick. You have a long ruler, but you can't remember that formula by that Greek fellow! However, you do have a pen and paper. What do you do? What if you lose the pen and paper, but you now have three identical bricks?



Medium 8. 36 students went to the zoo. 21 students liked the lions, 24 students liked the tigers, and 24 students liked the jaguars. 14 students like the lions and the tigers, 15 students liked the tigers and the jaguars, and 13 students liked the jaguars and the lions. One grumpy student didn't like any of the animals. How many students liked the lions and the tigers and the jaguars?



Medium 9. A stone to build the great pyramid of Giza is moved by rolling it on tree trunks, which are 2 metres in diameter? If the trunks make one complete revolution, how far does the stone move?



Medium 10. Brad and Angelina each have a handful of coins, with Angelina having one more coin than Brad. They both throw their coins on the table. What is the probability that Angelina obtains more heads than Brad?



HARD

Hard 1. Four horses enter the prestigious MathsMasters Cup. Assuming that there are no dead heats, how many different ways are there for the horses to cross the finish line? What if we include the possibility of dead heats?



Hard 2. The fictional blockbuster *Tropic of Calculus* has just been published. Its pages are numbered starting with page 1, and there are 2893 digits used to number all the pages. How many pages does the book have?



Hard 3. Start with the numbers 1, 1/2, 1/3, ..., 1/100. Choose any two numbers M and N from this list and replace them by the single number M + N + M x N. Repeat the process until only one number is left. What is that number?



Hard 4. What is the value of the following root monster?

$$\sqrt{22 + \sqrt[3]{22 + \sqrt{22 + \sqrt[3]{22 + \sqrt{22 + \sqrt{22 + \cdots}}}}}}$$

Hard 5. Burkard and Marty walk up a moving escalator. They start together on the bottom step of the escalator, and then Burkard takes two steps for each step that Marty takes. Burkard steps off the escalator on his 28th step, and Marty does so on his 21st step. How many steps of the escalator are visible at any one time?



Hard 6. At the MathsSnacks fast food outlet, you can order Kitty Nibbles in boxes of 6, 9 and 20. If you wanted exactly 25 Kitty Nibbles, for example, then no selection of boxes will satisfy you. What is the largest number of Kitty Nibbles such that no selection of boxes will give that exact number?



Hard 7. There are 100 playing cards on the tabletop, and exactly 23 cards are face up. Blindfolded, you want to separate the cards into two groups, each with the same number of cards facing up. How do you do it?



Hard 8. Ted and John play a game of placing \$2 coins on a flat plate. The coins cannot overlap, and the winner is the person to place the last coin to fit on the plate. Ted goes first. Who has a winning strategy, and what is it?



Hard 9. A castle is surrounded by a moat which is 20 metres wide. The brave Sir Isaac Newton wants to enter the castle, to rescue the fair maiden. He has only two planks, both of length 19 metres, and no nails. How can Sir Newton cross the moat?



Hard 10. Emmy Noether's car travels 120 km/h downhill, 80 km/h on level ground and 60 km/h uphill. It takes Emmy 6 hours to get from Functionville to Alegbraton, and 4 hours to return. How far is it between the two towns?



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